Abstract

A new approach, using stress functions, reveals how each component of the stress regime affects the stress pattern around the wellbore. The effect of tectonic far field stress on the stress trajectories in the host rock near a wellbore is visualized in a series of plots with the analytical stress trajectory solutions for a large range of net pressures on the wellbore. The deviatoric stresses around a wellbore result from the dynamic superposition of (1) far field tectonic stress, (2) near wellbore stress due to lithostatic pressure near the open hole, (3) pore over-pressure or under-pressure in the host rock, and (4) hydraulic pressure applied on the wellbore. The principal stress trajectory plots are used to determine the suitable options for well orientations and to delineate stress trajectory control of the incipient brittle failure patterns for hydrofracs and wellbore breakouts. Our approach provides fundamental insight, with an important practical application for improved understanding of the growth of hydrofractures.

Introduction

Drilling operations must be executed safer and cleaner to assure the concerned public that due diligence is exercised by oil and gas operators with a sustained effort to improve the drilling practice learning curve. In fact, drilling operations around the globe have been riskier than even the experts realized. A new analytical stress function description revealed that wellbores may unintentionally enlarge by concentric tension fractures in over-pressured rocks. Drilling engineers have long known that underpressured drill holes tend to collapse due to the pressure of the wall rock, which increases with depth. The pressure on the inside of the wellbore must be kept high enough to prevent the wellbore from collapsing inward (cf., Weijermars, 1998), which is why heavy bentonite mud is circulated down drill holes (cf., Achmed & Meehan, 2012).

Fracture caging is a recently recognized drilling hazard that directs induced fractures to curl around the wellbore (Fig 1). Fracture caging leads to cavitation/slabbing/spalling under specific physical conditions. The phenomenon has been described analytically (Weijermars, 2011) and examples appeared in numerical fully coupled fluid flow simulations (Zhang et al, 2011). The puzzling curling of hydro-fractures was presented independently by the two research teams (TUDelft, Netherlands and CSIRO, Australia) at the 45th US Rock Mechanics Symposium (ARMA) held in San Francisco, June 2011. One of the CSIRO numerical model runs (Zhang et al., 2011) confirmed that the circumferential tension fractures (Fig. 1) grow exactly as predicted by the analytical stress function theory (Weijermars, 2011) - the analytical and numerical results matched perfectly (Fig. 1).

Figure 1: Fractures 1 and 2 are, respectively, too short and long enough to escape from the fracture cage (red dotted ellipse) formed by the elliptical T1 - stress trajectories around the wellbore for an underbalanced bore hole, with negative net pressure P_NET = 1.66 times the largest far field compressive stress T1 (after Weijermars et al., submitted).
The present study further expands our work on hydraulic fracture growth by developing examples of stress trajectory solutions in natural tectonic settings in order to aid well engineers in optimizing drill hole safety, well integrity and productivity. Our results are equally valid for wells sunk in conventional, high pressure formations, such as occurring near salt bodies in the Gulf of Mexico, and for wells sunk in many of the deeper shales and tight gas plays.

Better well completions are particularly important for oil and gas production from shales, which benefit from an improved frac architecture that could enhance the well productivity. Today, a quarter of US shale gas wells are producing two-thirds of the gas from shales. That means the majority of the wells is either sunk in poor quality reservoirs or the hydraulic fractures did not tap effectively into the reservoir. Both the quality and safety of hydrofrac wells can be significantly improved by detailed understanding of the fracture caging mechanism and controlled engineering of the parameters to improve the hydraulic fracking process. The models outlined in this study show how safer and more accurate fracture placement can be achieved.

**Fracture caging**

The stress patterns around wellbores have traditionally been modeled using equations of Ernst Gustav Kirsch (1898), a 19th Century German engineer who described the elastic stress around a hole in an infinite plate under directional tension. The Kirsch equations have been adapted since several times (Malvern, 1969; Zoback, 2007) and can properly account for the stresses around so-called balanced boreholes, when the drilling fluid supports the inner wellbore with pressures that cancel out the any static pressures supplied on the outer wellbore. The typical break outs, tension fractures and stress trajectory patterns arising around such wellbores are shown in Figures 2a &b.

The stress function description of Weijermars (2011) captures the intricate interaction of any hydraulically induced stresses with the geological background stress and formation pressures, in a compact non-dimensional Frac number (Weijermars & Schulz-Ela, submitted):

\[
F = \frac{P_{NET}}{|T_1|} \quad (\text{Eq. 1})
\]

This study uses \( T_1 \) for the maximum compressive deviatoric stress (which is negative in our sign convention, and we use \( \sigma_1 \) for total stress, which includes the tectonic component of the confining pressure and which must be subtracted to arrive at \( T_1 \)).
The new insight added by this analytical approach is the recognition of a zone around the wellbore where induced fractures cannot escape (the ‘fracture cage’), which occurs when Frac numbers are negative (\( P_{NET} < 0 \)). Our sign convention makes dilational pressures positive and confining pressures negative. The wellbore net-pressure, \( P_{NET} \), is the unbalanced part of the wellbore pressure arising from the effective pressure due to the interaction of: (1) the lithostatic load on the wellbore of an open-hole completion, with or without drilling fluid, and/or (2) the differential between the hydraulic head of the drilling fluid and any initial pore pressure in the wellbore rock, and accounting for any tectonic residual pressure (Fig. 3a):

\[
P_{NET} = P_M + P_L + P_T + P_P
\]

(Eq. 2)

\( P_M \) is the hydrostatic load pressure on the wellhead, \( P_L \) is the lithostatic pressure on wellbore wall from rock overburden, \( P_T \) is the isostatic component of the tectonic stress and \( P_P \) is the unperturbed pore pressure in the formation.

The width of the fracture cage zone increases when the pressure of the penetrated formations rises. This can be mapped by characterizing the non-dimensional long axis \( E_L^* \) of the fracture cage ellipse as a function of \( F \) (Weijermars et al., submitted):

\[
E_L^* = \sqrt{\left[\frac{3 (1 - 2F^2)}{2} + \sqrt{(3F^2 - 12F - 3)}\right]/2},
\]

for \( F < -0.45 \)  

(Eq. 3)

This equation gives the non-dimensional distance \( E_L^*(=E_L/a) \), measured from the borehole center to the neutral point (Fig. 3b). Figure 4 shows that the fracture cage area grows when the magnitude of \( |P_{NET}| \) increases over 1.

In fact, over-pressured rocks are well known to be hazardous when drilled. The 2010 Macondo (Gulf of Mexico) and the 2006 Lapindo (Indonesia) well blowouts (DHSG, 2011; Davies et al., 2010) more than likely resulted from underbalanced wellbores entering over-pressured formations. The next sections systematically outline practical situations that may arise.

### Balancing wellbore pressures

When planning a wellbore orientation involving well stimulation by hydraulic fracturing, the orientation of the wellbore must be positioned such that optimum synergy is achieved with the stress pattern around the wellbore. The well must be engineered such that stress patterns are utilized and manipulated in a controlled fashion. This requires not only accurate estimates of the far field stress orientation and magnitude in the tectonic block entered by the wellbore, but also detailed knowledge (or estimates) of the wellbore pressure components \( P_M, P_L, P_T, \) and \( P_P \), which determine \( P_{NET} \) as per equation (2). The lithostatic pressure component, \( P_L \), is a function of depth and can be accurately estimated for any well:

\[
P_L = \frac{(1 + \nu)/(3 - 3\nu)}{\rho g h}
\]

(Eq. 4a)

with specific density \( \rho \), gravity \( g \), depth \( h \) and Poisson

---

**Figure 4**: Dimension of the fracture cage longest ellipse axis \( E_L^*(=E_L/a) \), as a function of the Fract number \( F \) (based on Eq. 3).
ratio $v$. The tectonic pressure component, $P_T$, is a function of tectonic stress and can be estimated from the state of stress in the crustal section entered by the well:

$$P_T = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3} \quad \text{(Eq. 4b)}$$

The pore pressure component, $P_P$, is determined by the compaction and burial history of the basin, and may result in severely overpressured sedimentary formations. Salt deposits provide a particularly challenging environment, as their Newtonian viscosity means these rocks cannot sustain any shear stresses and the pressure in the salt formation therefore is due to the average differential load on the entire salt body:

$$P_P = \Delta \rho g \Delta h \quad \text{(Eq. 4c)}$$

The drilling mud pressure or hydraulically applied pressure, $P_M$, is the only external controllable, which must be cleverly manipulated to engineer the state of stress around the wellbore.

First we assume a vertical wellbore that penetrates a synthetic sedimentary basin (Fig. 5a). The initial set-up assumes no tectonic background stress, the lithospheric slab is in a state of static equilibrium. The stress trajectories around the wellbore will be typically spiderweb patterns, either underpressured or overpressured (Fig. 5c&d), depending on the value of $P_{NET}$. For example, if $P_{NET} = 0$, then no

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**Basic Tectonic Settings**

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<thead>
<tr>
<th>a) Tectonic Stress</th>
<th>b) Tectonic Compression</th>
<th>c) Tectonic Tension</th>
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<td>Isotropic</td>
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<th>d) Strike Slip</th>
<th>e) Transpression</th>
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**Figure 6a:** Overbalanced wellbore with radial planes of preferred tensional frac failure normal to the least principal stress $T_3$. Breakout cannot occur, due to unfavorable shear stress orientations. **6b:** Underbalanced wellbore with cylindrical surfaces of preferred tensional failure, due to a reversal of the principal stresses. Breakouts may occur in all radial directions, due to favorable shear stress orientations.

**Figure 7a-f:** Basic tectonic settings for a sedimentary basin. The principal stress orientations on the boundaries of the central deformation zone are determined by the horizontal motion vectors of the crustal segments adjacent to the basin in between (after Weijermars, 1993).
deviatoric stress is imposed on the rock around the wellbore and principal stress trajectories vanish altogether. The stress trajectories for the cases of \(P_{NET} > 0\) and \(P_{NET} < 0\) in a borehole with radius ‘\(a\’) (Figs. 5b &e) are given by the following stress function (Weijermars, 2011):

\[
\Phi(r, \theta) = P_{NET} a^2 \ln R
\]  
(Eq. 5)

The tensor components of the radial, tangential and shear stresses are given by (valid only for \(R \geq a\)):

\[
T_R = P_{NET} (a/R)^2 \quad \text{(Eq. 6a)}
\]

\[
T_\theta = -P_{NET} (a/R)^2 \quad \text{(Eq. 6b)}
\]

\[
T_{\theta\theta} = 0 \quad \text{(Eq. 6c)}
\]

Although the shear stress tensor component is zero for the chosen polar reference frame, shear stresses do occur in surfaces oriented obliquely to the tangential and radial stress components. Figures 6a & b show the preferential hydraulic fractures for an over pressured and under pressured wellbore, respectively, based on the stress trajectory solutions of Figures 5a&b. One should realize that along a single wellbore both types of frac patterns may occur and alternate in subsequent layers, due to variations in the formation pressure, which may render \(P_{NET}\) either smaller or larger than zero along individual sections of the wellbore.

**Basic tectonic settings**

Sedimentary basins containing hydrocarbon provinces may be located in zones of active tectonic movement (Fig. 7a-f). The Andersonian stress states (with deviatoric stresses \(T_1 = 0\)) are given in Figs 7b-d. The cases for transpression stresses (\(T_2\) is extensional, Fig 7e) and transtension (\(T_2\) is compressional, Fig. 7f) arise when a strike slip motion (Fig. 7d) is superposed on a tectonic compression (Fig. 7b) or tension (Fig. 7c). Expressions for the relationship between the crustal movement vectors and the deviatoric stresses on the boundaries of the central deformation zone have been comprehensively modeled elsewhere (Weijermars, 1993).

The standard procedure in hydraulic fracking is to drill in the direction of the least principal stress (Fig. 8a), such that frac planes will open perpendicular to \(T_3\) in the \(T_1-T_2\) planes (cf. Zoback, 2007). However, this is impractical for collision zones, as these have the least principal stress vertical (Fig. 7b). For shortening basins, this approach would limit suitable well orientations to vertical boreholes (Fig. 8b). An alternative well orientation is suggested here, using the more economic open-hole completions that work in synergy with the tectonic stress field even in compressional zones (Fig. 8c).

*Figure 8a:* Standard well orientation for fracking horizontal layers with horizontal wellbores aligned with the least principal tectonic stress direction is not suitable for the tectonic stress regimes in compressional basins. *8b:* Compressional tectonic settings would then require a vertical wellbore orientation, which may be impractical if a thin horizontal layer needs to be fracked for lateral production access. *8c:* Alternative well orientation suggested for open-hole frac stages that work in synergy with the tectonic stress field in compressional basins, and suitable for thin production zones.
A more refined understanding of potential frac patterns is possible when the effect of $P_{\text{NET}}$ on the principal stress trajectories is taken into account. Figures 9a-f show the stress trajectories that arise for various net pressures relative to the tectonic background stress, as expressed by the Frac number (cf. Eq. 1). The stress trajectory pattern for $F=1$ (Fig. 9a) corresponds to breakouts and tension fracture orientations that are still close to those familiar from the classical solutions portrayed in Figs. 2a&b (which occur for $F=0$). However, higher wellbore net pressures ($P_{\text{NET}}$) relative to the tectonic stress and as expressed by the Frac number, lead to stress trajectory patterns that progressively change toward a spiderweb pattern, with the extreme case occurring for $F=+\infty$ (Fig. 9f). Such overbalanced wellbores will stimulate radial tension fractures fanning out from a central axis along the longitude of the wellbore. Breakouts can not occur for high $F$ number conditions. The potential frac patterns delineated in Figs. 9a-f can be engineered by manipulating the Frac number appropriately by hydraulic stimulation.

A completion method proposed here for thicker production layers (e.g., Vienna gas shales) in compressional basins makes use of multilaterals to zipper frack the zones in between the horizontal wellbores (Fig. 10), using high Frack number fracking pressures. Although the condition of a rock continuum is assumed in our stress function model approach, this completion method for optimized rock volume access is also recommended for coal bed methane recovery in compressional basins.
Figure 11a: Sediments in extensional basins can be fracked with the standard well orientation of a horizontal wellbore aligned with the least principal tectonic stress direction. 11b & c: Alternative well orientations suggested for open-hole longitudinal frac stages that work in synergy with the tectonic stress field in extensional tectonic settings.

Figure 12a-f: Preferred frac patterns (black curves) and breakouts (red cusps) around overbalanced wellbores. Fracs follow $T_1$ stress trajectories and are scaled by the Frac number indicated. The horizontal is aligned with the far field extension axis $T_3$, and the wellbore points toward $T_2$, which is normal to the plane of view.
Extensional zones (Fig. 7c) can be stimulated by placing transverse fracs along horizontal wellbores aligned with the least principal stress $T_3$ (Fig. 11a). Alternative well architecture strategy uses vertical wellbores Fig. 11b) or aligns the horizontal wellbore with the trend of the extensional graben to place longitudinal frac stages in the $T_1-T_2$ plane normal to $T_3$ (Fig. 11c). If $P_{NET} > 0$, then the stresses around the horizontal wellbore in the extensional stress regime assume the stress trajectory patterns outlined in Figs. 12a-f. The corresponding frac patterns and breakouts are also delineated. Higher Frac numbers open wider fans of frac surfaces (Fig. 12f). This knowledge favors the emplacement of longitudinal frac fans along horizontal wellbores by rendering $P_{NET}$ higher using a high hydraulically applied $P_M$ (see Eq. 2). The longitudinal fracs of Fig. 11c can be engineered to develop fan shapes as delineated in the sections of Figs 12a-f, and controlled by the $F$ number. The productivity from the fanning fracs is likely to be higher than can be achieved with any transverse frac architecture. This would favor using longitudinal open-hole fracs (Fig. 11c) over transverse fracs (Fig. 11a).

A third suite of tectonic settings occurs when sedimentary basins are situated in strike-slip shear zones, transpression or transtension zones (Figs. 7d-f), which have the $T_1-T_2$ plane oblique to the basin boundary. Horizontal layers in such basins can be fracked by a range of wellbore orientations, including the array of Figure 13.

**Overpressured formations**

The state of stress around wellbores that enter overpressured formations is very different from those shown in the previous section. Overpressured rocks exert an inward pressure on the outer wellbore, and if not balanced this will result in an underpressure in the borehole. The Frac number for such bore holes is negative, as $P_{NET} < 0$. In fact, this situation is very common as it is a prerequisite for maintaining net flow into any wellbore that produces from a hydrocarbon-bearing...
formation. The unique feature of negative Frac number wellbores is the occurrence of a fracture cage zone around the wellbore (Weijermars et al., submitted).

Figures 14a-f show the full range of principal stress trajectories around wellbores characterized by negative Frac number conditions. The elliptical tension fracs outlined near the wellbore are all inside the fracture cage zone. The placement of fracs in overpressured formations is severely handicapped by the occurrence of these fracture cages, as fracs cannot propagate outward. What is further critical is that such wellbores are intrinsically unstable due to a high risk of cavitation with spalling and breakouts. The increased risk of blowouts and well integrity loss in underbalanced drilling has been recognized before. However, we assert this is a systematic hazard, and the mechanism is concisely captured by the $F$ number, which quantifies the width of the fracture cage around underbalanced wellbores. The serious risk of a run-away effect by repeated breakouts and spalling fractures along the circumference of underpressured boreholes is particularly high when drilling into overpressured formations. The wellbore at depth may progressively widen and the upward flow of rock fragments removes the debris from the overpressured formation. New fracture cages will form around the widened wellbore as long as the formation overpressure maintains the flow into the underpressured wellbore. The mechanism of runaway fracture caging will not stop until the formation overpressure has been released to values below the tensional strength of the wellbore rock, unless stopped by pressure control devices. This means that fracking using pistons that apply the hydraulic pressure at confined injections points along the borehole liner (King, 2010) may further enhance the fracture caging effect. Tension fractures may only propagate outward from the fracture cage when an initial fracture is long enough to transfer the hydraulic pressure into the wall rock where it stops until the formation overpressure has been released to values below the tensional strength of the wellbore rock, unless stopped by pressure control devices. This means that fracking using pistons that apply the hydraulic pressure at confined injections points along the borehole liner (King, 2010) may further enhance the fracture caging effect. Tension fractures may only propagate outward from the fracture cage when an initial fracture is long enough to transfer the hydraulic pressure across the fracture cage zone into the wall rock where $T_1$ - trajectories are no longer forming closed ellipses [see Fig. 1 for principle, and Weijermars et al. (submitted) for further details].

Discussion and Conclusions

The optimum solution for frac architecture is no longer an arbitrary outcome (Fig. 15) when using the principles of wellbore orientation in synergy with the tectonic stress regime and engineering $P_{NET}$ such that the fracs will develop as desired. Far field tectonic stresses may range between 0 and 1,000 MPa (Zoback, 2007). In unbalanced open-hole completions, the wellbore pressure $P_{NET}$ may rapidly turn negative due to the contribution of $P_T$ to $P_{NET}$ as accounted for in Eqs (4b) and (2), respectively. The lithostatic pressure component also increases rapidly with depth (25 MPa/km, c.f., Weijermars, 1998). Negative Frac number conditions prevail in such wellbores, and the stress trajectory patterns of Figs. 14a-f are applicable for such cases. If the hydraulic pressure and well head load can be raised high enough to render $P_{NET}$ positive, the stress trajectory patterns of Figs. 9a-f and 11a-f are valid for such cases.

In any frac job, there may be interference from natural fractures, which must be taken into account using additional dual porosity flow models to estimate how the well productivity can be stimulated. However, knowing the ideal frac architecture in continuous rocks for the basic tectonic settings as systematically outlined in this study will help to develop realistic and reliable hydrocarbon flow models that account for fracture highways in hydrocarbon producing (shale) basins.

The generic tectonic settings outlined here can be translated to specific geographic locations and basin settings. Further modeling is required for such basin plays to determine the detailed stress trajectory patterns in the crustal region to plan for proper well orientation and develop strategy options for optimum frac placement. This is especially important in tectonically active and faulted basins where the principal stresses may rotate across discrete surfaces and terminate against isotropic salt masses (Fig. 16). The brittle-ductile transition across salt layers has been scaled in earlier work (Weijermars et al., 1993) as well as the refraction of principal stresses across anisotropic layers (Weijermars, 1992). The inferred stress trajectories in Figures 16 are typical for extensional zones in North Sea basin grabens with buoyant salt pillows. Our aim is to develop detailed stress trajectory profiles and maps for specific basin settings, using a variety of analytical and numerical modeling techniques at various scales, integrated with seismic verification techniques. This approach will help to plan for safe and effective wellbore orientations and design of field development solutions with optimized frac architecture.
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References
Weijermars, R., & Schultz-Ela, D., Submitted 2012. Frac Number and Hydrofrac Nomogram for Continuum Stress Trajectory Solutions around Wellbores.