Shale gas technology innovation rate impact on economic Base Case – Scenario model benchmarks

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Abstract

Low gas wellhead prices in North America have put its shale gas industry under high competitive pressure. Rapid technology innovation can help companies to improve the economic performance of shale gas fields. Cash flow models are paramount for setting effective production and technology innovation targets to achieve positive returns on investment in all global shale gas plays. Future cash flow of a well (or cluster of wells) may either improve further or deteriorate, depending on: (1) the regional volatility in gas prices at the wellhead – which must pay for the gas resource extraction, and (2) the cost and effectiveness of the well technology used. Gas price is an externality and cannot be controlled by individual companies, but well technology cost can be reduced while improving production output. We assume two plausible scenarios for well technology innovation and model the return on investment while checking against sensitivity to gas price volatility. It appears well technology innovation – if paced fast enough – can fully redeem the negative impact of gas price decline on shale well profits, and the required rates are quantified in our sensitivity analysis.

1. Introduction

There is a growing interest in the assessment of the world’s shale gas resource potential since a groundbreaking global inventory by Rogner [1]. Many regional exploration efforts are underway to establish the presence and volume of prospective natural gas resources [2]. The development of unconventional hydrocarbon fields in shale gas provinces remains economically risky, because the estimated ultimate recovery (EUR) often remains poorly constrained – especially during the early stages of the play development [3,4]. Subsurface uncertainties related to geological conditions are particularly high in unconventional gas fields due to the lack of hydrocarbon fluid interconnectedness between adjacent wells. Each well is a new reservoir itself and the intrinsic geological uncertainty causes a large spread in well flow rates, even between adjacent wells. Consequently, EUR volumes, indicative for the return on investment, may vary greatly per well within unconventional gas fields [3,4].

In any emerging shale gas play, the volume of oil and gas resources in place (OGIP) must first be appraised, which then leads to estimations of technically recoverable resources (TRR; Fig. 1a).
The fraction of TRR that remains unrecovered due to unfavourable economics has been coined the economic gap factor, $EG$ [5]:

$$EG = TF - RF \tag{1}$$

with technology factor $TF$ and recovery factor $RF$ defined as follows. The fraction of the estimated initial oil and gas resource in place (OGIP) that can be ultimately booked as a proved reserve (the EUR volume) is determined by the realized recovery factor, $RF$ [6]:

$$RF = EUR/OGIP \tag{2}$$

The volumetric proportion of OGIP that can be classified as TRR is determined by the technology factor, $TF$ [5]:

$$TF = TRR/OGIP \tag{3}$$

$TF$ increases over time as new reliable technology will be made available through research and development (Fig. 1). For example, technology advances like hydraulic fracturing and horizontal drilling have increased the $TF$ for shale gas plays. As $TF$ increases TRR grows, but technology efficiency sets an upper limit for the final recovery factor, $RF$. The fraction of TRR remaining undeveloped due to any technology being incapable of extracting more gas (from the established OGIP) is given by $1 - (RF/TF)$. At a certain gas price and extraction cost, only a fraction of the TRR turns out to have a profit potential, which are termed the economic recoverable resources (ERR).

To put above terms into perspective a brief practical example is merited. The OGIP for the US Barnett shale play has been recently estimated at 444 Tcf [7]. As of 2012, already 13 Tcf was recovered [8], which implies a recovery factor $RF = 2.9\%$. However, better technology and better well deployment choices (multilateral horizontal wells, precision hydraulic fracturing and finding natural fracture fairways) have lead to cost efficiency, and has brought down shale gas break-even cost. Estimates of the final EUR for the Barnett shale when depleted in 2050 range between 26.7 Tcf (Low Case [8]) and 45.1 Tcf (Base Case [8]), which means its $RF$ ranges between 6 and 10\%. An earlier study of the Barnett estimated TRR for the Barnett to amount to 44 Tcf [6], which means $TF = 10\%$. Other estimates of $TF$ for a range of shale gas plays range between 15\% and 40\% [9], but these are highly speculative for relatively under-investigated shale plays. If the final $RF$ for the Barnett shale play will eventually be managed such that $RF$ equals $TF$ (i.e., 10\%) all of its technically recoverable gas will have been extracted (by 2050 [8]).

One must bear in mind that in addition to technology, economic factors may limit shale gas development. The volume of gas that can be extracted economically gives the so-called economically recoverable resources (ERR). The ERR may fluctuate with gas prices for any cost of current technology and $ERR < TRR$ at anyone time (Fig. 1). Gas demand and gas market prices are the most important (but volatile) determinants for which portion of TRR will actually be economically recoverable. Prevailing gas prices in the US have been so low in the period 2008–2013, that the economic gap between TRR and ERR has been growing [10–13]. To reverse the trend and close the gap, finding, development, and completion, costs must come down and gas prices must go up. Over time $TF/RF$ should ideally converge to 1, otherwise technically recoverable resources remain undeveloped. Other factors influencing ERR include proximity to gas transmission and delivery infrastructure, environmental legislation, permitting speed, population density, and degree of public support [14].

There are several ways to increase the output volume and longevity of production from regions with shale TRR in place. The principal drivers for reserves growth are (Fig. 2): (a) higher wellhead gas prices due to increased demand (or tight supply), and (b) lower production cost due to technology innovation (and some aid by favorable taxation policies). Both drivers can reduce the economic gap and ideally $RF$ and $TF$ become equal (Eq. (1)). Technology generally improves over time and efficiency gains reduce cost so that ERR grows. The EUR cannot exceed ERR, because only economic resources may ultimately classify as reserves (Fig. 1b). Fastest growth of ERR can be realized when the gas wellhead price, taxation policies and technology innovations all develop favorably. For example, federal or regional government decisions for favorable taxation and royalty policies can help shale gas companies to unlock new reserves from known, technically recoverable resources.

As long as gas prices are uncontrollable and shale gas operators must turn a profit from their assets at prevailing wellhead prices, technology innovation is a key factor which can help to keep operations profitable even when gas prices fall. This study models the effect of well technology innovations which can effectively raise the EUR while simultaneously lowering the cost per well. The critical impact of such well technology innovations is a reduction of capital expenditure and a boost of well productivity at the same time. The well technology innovation scenarios considered below are supported by historic well performance improvements reported by SW Energy (an independent US shale gas company), which has halved drill time and doubled the initial well productivity over a five year period (Fig. 3). The doubling of the well productivity can be attributed mostly to the increase in length of the average wellbore, which has more than doubled over the indicated period.

The cash flow models developed below demonstrate what rate of technology innovation is needed to turn out positive earnings when gas prices are weakening. In essence, overall shale gas
production rate and revenue must remain high enough to cover all cost and still leave a margin to compensate investors in the company commensurable with risk. Each shale play is unique and the response of production to price may be quite different from play to play. Field development plans for shale gas assets need to be validated using improved models for estimating well productivity, price volatility and field development costs to ensure cash flow will remain positive [5,13]. Operational profits are presently not materializing for a large number of US shale gas operators [17]. A strong focus on technology innovation rates remains important to restore profits for shale gas operations.

2. Methodology

The models documented in this study are based on well productivity decline functions and discounted cash flow equations. The relevant algorithms are incorporated in a proprietary Excel-based interface developed by Alboran Energy Strategy Consultants, which enables the calculation of the field development scenarios and was used to produce the plots in the present study. A concise manual is made available as a complimentary resource in an online repository [18].

Emerging shale gas plays typically have a high degree of subsurface uncertainty due to which field development in the early stage inevitably includes wells with a lower productivity and marginal cash flow. The mean EUR/well improves when the subsurface model of an emerging shale gas play becomes more reliable over time. Uncertainty is reduced because the well population grows and brings in more data as operations zoom in on the so-called sweet spots of a developing shale gas play.

Further gains are possible when well technology innovation lowers drilling, completion and fracking cost. New and cheaper technology is needed to develop each well more effectively. Cash flow analysis can be used to assess the rate of technology improvement required to turn shale gas plays into positive net cash flow territories. Table 1 shows the typical input values used for a cash flow simulation for a typical shale gas field development project where wells are drilled with Base Case Technology. The cumulative gas production of a Base Case Well is here assumed to be 2 EUR, which is representative for wells in the best areas of the Barnett and Fayetteville assessment units [19].

2.1. Well productivity Base Case

Understanding the well productivity of representative US shale gas plays provides important guidance for the economic development of shale gas wells in emergent shale plays elsewhere in the world. In the Barnett shale play, estimates for the mean EUR for representative horizontal wells range between 1.0 and 1.6 bcf/well [19–21], and there is considerable spread in well performance for subareas. In the ‘best areas’ for the Barnett a representative mean EUR is 2.1 bcf/well, and the ‘worst areas’ have a mean EUR of 0.59 bcf/well [20]. For this study, we adopt a Base Case output of 2 EUR/well, which is representative for wells in the best areas of the Barnett and Fayetteville assessment units [19]. We assume the well EUR can be modeled by an exponential decline function:

$$q_n = q_i (1 + a)^n$$

(4)

$q_n$ is the flow rate in year $n$, $q_i$ the starting flow rate in first year, and 'a' the annual decline rate (remember this is a negative fraction), and 'n' the number of years. Hyperbolic decline functions can also
be considered, but have the added drawback that wells would continue to produce indefinitely [22], which is physically impossible. Still more complex well behavior has been suggested [23] and is likely to be valid but we use an exponential Base Case well for its transparency and convenience for demonstration of principle.

Fig. 4a plots the annual well productivity over a 25 year lifecycle \( (n = 1, \ldots, 25) \) using the gas production from a single Base Case well (Table 1). The well flow rates of Fig. 4a were obtained by adopting initial productivities \( q_i = 0.3 \text{ bcf/y} \) in year 1 and a decline factor of 15\% \((-0.15)\) fitting the Base Case well with an appropriate decline function as per Eq. (4). This gives over a 25 year lifecycle \( (n = 1, \ldots, 25) \) of a Base Case shale gas well a cumulative production of 2 bcf (Fig. 4b), which is the average EUR/well for Base Case Technology.

The annual production for a field development project consisting of 100 wells drilled with Base Case Technology in the first decade at a rate of 10 wells per year is given in Fig. 4c. The annual well output increases with the number of wells, but abrupt decline sets in when the drilling is stopped in year 10 (Fig. 4c). The gas output of the aggregate wells declines over the remaining 15 years of

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<tr>
<th>Scenario options</th>
<th>Standard technology</th>
<th>Well technology innovation scenarios</th>
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<tr>
<td>Technology used</td>
<td>Base Case Well</td>
<td>Well Technology A</td>
</tr>
<tr>
<td>EUR/well over 25 Yrs</td>
<td>2(^a)</td>
<td>3(^b)</td>
</tr>
<tr>
<td>Well CAPEX ($/MM)</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>OPEX ($/Mcf)</td>
<td>1.2</td>
<td>1.2</td>
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<tr>
<td>Other CAPEX ($/Mcf)</td>
<td>0.3</td>
<td>0.3</td>
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\(^a\) Average for modern ‘better’ horizontal wells, \( q_i = 0.3 \text{ bcf/y}. \)
\(^b\) \( q_i = 0.45 \text{ bcf/y}. \)
\(^c\) \( q_i = 0.6 \text{ bcf/y}. \)

![Fig. 4. (a) Production profiles for single gas wells (Base Case Technology) with initial production rates \( q_i = 0.3 \text{ bcf/y} \) and a decline factor of 15\% \((-0.15)\) fitting the Base Case well with an appropriate](image-url)
2.2. Well technology innovation

We now consider the effect of technology innovation on well productivity and field performance. **Well Technology A** assumes a 50% EUR enhancement over the Base Case well, and the average CAPEX/well is reduced by 25% over the well’s life cycle (Table 1). **Well Technology B** assumes a 100% EUR enhancement over the Base Case well, and well CAPEX is halved as compared to the Base Case (Table 1).

Fig. 5a shows an improvement of the production rate over the Base Case well rate due to introduction of Well Technology A with an initial productivity of $q_i = 0.45 \text{ bcf/y}$, giving a total EUR/well of 3 bcf (Fig. 5b). Well Technology B effectuates an additional improvement in well productivity (additional ‘learning’), which raises the mean EUR per well to 4 bcf (not separately included here for space conservation). Fig. 5c plots the total annual production for a shale field project with 100 wells drilled over a decade with Well Technology B. The cumulative production of the field development project is raised, by using Well Technology B, to an EUR of 400 bcf (Fig. 5d) as compared to less than 200 bcf with Base Case Technology (Fig. 4d). This improved EUR required the following input parameters: $q_i = 0.6 \text{ bcf/y}$ and the decline rate $a$ is kept at $-0.15$ (Table 1 input using Eq. (4)).

2.3. Economic model

In order to upgrade the model of well productivities and cumulative production (developed in Sections 2.1 and 2.2) into an economic analysis, cost factors need to be considered. Table 1 shows the typical values used in our cash flow model simulation for CAPEX, OPEX, royalties, tax rates, depreciation and discount rates, all of which affect the outcome of a cash flow model for a specific field asset. The outlay of capital expenditure (CAPEX) and operating expenditure (OPEX) can be controlled by the operator. The taxes and royalties due are mostly controlled by the governing petroleum extraction laws and rules. Appendix A provides a brief set of algorithms used for the cash flow analysis in our study.

For the gas price development an inflation function is adopted with initial gas price set at $4/\text{Mcf}$ and a typical annual inflation factor of 2.5% (Fig. 6):

\[
p_n = p_0(1 + b)^n
\]

$p_n$ is the price in year $n$, $p_0$ is the wellhead price in first year, and $b$ is the annual inflation rate affecting the gas price, and $n$ the number of years. The gas price development is largely an external uncertainty, sometimes affected by policy measures. The overall economic margin on the production of proved reserves remains critically vulnerable to volatility in wellhead prices. In our models 1000 cubic ft (1 Mcf) of gas is equivalent to a calorific value of 1 million British thermal units (1 Mmbtu) used in market pricing.

\footnote{Use of units: Imperial units still dominate the professional literature on oil and gas projects, because US oil industry uses such units. European companies use both Imperial and metric units. This study uses Imperial units. Brief conversion rules are given here: 1000 cubic feet $\approx$ 28.32 cubic meters; 1 cubic meter $\approx$ 35.31 cubic feet; Prices of natural gas are given in $/\text{Mmbtu}$, which approximates $/\text{GJ}$ or $/\text{Mcf}$; the calorific value of 1 Mmbtu approximately equals 1 GJ; for standard gas 1 Mmbtu $\approx$ 1GJ $\approx$ 1Mcf $= 1000$ cubic feet. In our study 1000 cubic ft (1 Mcf) of gas is equivalent to a calorific value of 1 million British thermal units (1 Mmbtu) used in market pricing.}
In the below analysis we assume a classic appraisal method of project net present value (NPV) and profitability (IRR) were externalities that companies are not required to account for are excluded. The exclusion of such external costs (possible road damage, decline in residential property value, health impacts and global warming due to methane emissions) are cause of societal concern and subject of the ongoing worldwide debate on the benefits of shale gas development (see discussion in Section 4).

3. Results

3.1. Shale field cash flow – Base Case wells

Fig. 7a plots the undiscounted cash flow for our Base Case field project with 100 wells drilled over a decade, and with cumulative gas production output based on the Base Case well performance of Fig. 4d. Economic analyses of the corporate performance of gas companies typically use a large numbers of consolidated wells. The model does not account for any failed wells. Fig. 7b shows the cumulative undiscounted cash flow for Base Case wells. The gas price assumed is $4/Mcf initially, but a full range of gas prices is introduced in a later sensitivity analysis. For a gas price of $4/Mcf and gas price inflation correction, the internal rate of return (IRR) of a shale field developed with Base Case Technology is precisely 0, and the net present value (NPV) of the project equals 0. There is no financial room for the SEC-mandatory 10% discounting, as the NPV would turn negative. Clearly a gas price of $4/Mcf would not result in a profitable field project when Base Case wells are used.

The impact on Base Case wells of higher initial gas prices is shown in Fig. 7c-f. If the wellhead price is increased to $6/Mcf, the field project with an array of 100 Base Case wells still has a meager IRR of 14% (Fig. 7d). However, if the wellhead price rises further to $8/Mcf, the Base Case wells yield a positive cash flow with an attractive IRR of 30% (Fig. 7f). The sensitivity of the IRR of the shale field project with Base Case wells to gas price fluctuations is summarized in Fig. 8. For a gas price below $6/Mcf the corporate hurdle rate (commonly at 15% or higher) is not satisfied from the gas production sales by the Base Case wells. Heavy losses are incurred when the gas prices drop below $4/Mcf (Fig. 8).

3.2. Impact of technology innovation

Field development with Well Technology A or Well Technology B rather than Base Case Wells improves both the NPV and IRR of the shale field project. Fig. 9a–d shows the cash flow models that quantify the economic impact of Well Technology A and B (using input data of Table 1). We first assumed an initial US gas price of $4/Mcf, which appreciates by a fixed inflation compensation (Eq. (5)). Our cash flow models demonstrate that field development with Well Technology A (Fig. 9a) rather than Base Case Wells (Fig. 7a) improves the NPV. The cash flow effect of using Well Technology A improves the NPV of the field (NPV = $200 million for a gas price of $4/Mcf; Fig. 9b) and the effect is comparable to an initial gas price assumption of about $5/Mcf using the Base Case wells (Base Case NPV = 0 for $4/Mcf – Fig. 7b; Base Case NPV = $300 million for $6/Mcf – Fig. 7d). The adoption of Well Technology B (Fig. 9c) has an even larger effect on project NPV. The effect on cash flow of using Well Technology B (NPV = $600 million, Fig. 9d) at $4/Mcf gas price, is equal to the effect of a price hike to $8/Mcf using Base Case wells only (NPV = $600 million at $8/Mcf gas price, Fig. 7f). Clearly, the assumed rate of technology innovation can make any US shale gas operation profitable, even at $4/Mcf.

3.3. Summary sensitivity analysis

The sensitivity of the profitability of the shale project in terms of internal rate of return (IRR) was systematically investigated, using technology innovations as well as gas price volatility as variables (Fig. 10). Field development with Well Technologies A and B rather than Base Case Wells improves the IRR. The IRR of Well Technology B for an initial gas price assumption of $3/Mcf (IRR = 14%) is comparable to that of $6/Mcf using Base Case wells (Fig. 10). At a gas price of $4/Mcf it appears Well Technologies A and B can both redeem the IRR loss that would occur when Base Case wells are used (Fig. 10). Our sensitivity analysis of Fig. 10 proofs that better technology and better well deployment choices may lead to cost efficiency that can bring down shale gas break-even cost while locating sweet spots. The rate of technology innovation used as input parameters in Table 1 was inspired by real achievements as summarized in Fig. 3. Our method can now be used to set firmer targets for the rate of technology improvement that could make shale gas fields profitable again.

4. Discussion

The worldwide pursuit of shale gas development requires the localization of hydrocarbon resources followed by a systematic assessment of the economic production potential. Shale gas operators must zoom in on leads, prospective resources and then proceed to detect sweet spots that provide the attractive EUR for proved reserves. Well performance metrics and cost control together with tax liabilities determine the EUR growth rate (Fig. 1). Lower technology cost due to smarter well completions and better well deployment choices may lead to production increases and reserve additions.

Although reserves tend to grow over time, the early phases of geological resource assessment involve large uncertainty as the required subsurface data (well logs, petrophysics, hydrocarbon maturity, fracture patterns and seismic sections) are still scarce before a shale play is fully developed. For example, estimates of the Barnett’s proved reserves were lowered by the EIA by 8.9 Tcf to 23.7 Tcf between 2011 and 2012 [24]. The assessment by the Bureau of Economic Geology (UT Austin) maintains EUR estimates for the Barnett ranging between 26.7 Tcf (low case) and 45.1 Tcf (high case) with anticipated depletion by 2050 [8]. Although the low scenario cannot be excluded according to two independent studies [25,26], technology gains and gas price development will determine whether the low or high case will be realized by 2050 (or beyond).

Section 2.3 stated that the economic model used in this study follows a discounted cash flow model where company expenses
adhere to the traditional project appraisal method. Externalities that companies are not required to account for are costs associated with operational road wear, decline in residential property value and possible health impact related to drilling fumes, reduced air quality and deterioration of aquifers. Another, increasingly debated aspect is the impact of methane induced climate change [27–29].

Fig. 7. (a) Cash flow model for shale gas field developed with Base Case well technology (100 wells; initial flow rate at 0.3 bcf/y per well), with an initial gas price of $4/Mcf. (b) Cumulative undiscounted cash flow is zero for this case. (c) Cash flow model for Base Case (100 wells) using a higher initial gas price of $6/Mcf. (d) Cumulative cash flow for this case gives and NPV = $314 million and IRR = 14% (both before discounting, but including 10% depreciation), and payback after 14 years. (e) Cash flow model for Base Case (100 wells) using a gas price of $8/Mcf. (f) Cumulative cash flow (10% discount rate) gives an NPV = $626 million, IRR = 30%, and payback after 8 years. Discounted net cash flow shows depreciated NPV further lowered by 10% discount.
which is externalized to the public commons (as in the “Tragedy of Commons” [30], c.f. [31]). Industry advocates would allegedly overstate the economic benefits [32], but these include employment opportunities, development of new transportation infrastructure and water supply systems in remote regions that can bring lasting solutions for the local communities and farmers, as well as capitalization of resource endowments as tax revenues. We agree that input–output economics should be considering all externalities [33]. The industry is under pressure to counter public concerns about stated benefits versus negative impacts, proving whether these are perceived or real, will establish the outcome of the societal debate. Industry therefore should welcome regulation aimed at eliminating all pollutants where inadequate policies and breach of compliance occur. Global introduction of such regulation and improved policies will avoid economic distortion and improve compliance, provided proper audits and penalties are enforced to counter breach of compliance. The exclusion of costs of certain externalities could be interpreted as an unfair tendency for socializing expenses and privatizing profits.

Fig. 8. Sensitivity of IRR for gas production using Base Case well (all input parameters specified in Table 1). At $4/Mcf, the Base Case well IRR is exactly 0 and NPV = 0 (see Fig. 7b). At $6/Mcf the Base Case well IRR = 14% (Fig. 7d). For $8/Mcf gas price, the IRR is a handsome 30% (Fig. 7f).

Fig. 9. (a) Cash flow model for Well Technology A (initial flow rate at 0.45 bcf/y per well, Table 1) in a shale gas field with 100 wells, assuming an initial gas price of $4/Mcf. (b) Cumulative cash flow (10% discounted rate) gives a project NPV = $181 million and IRR = 10% (both before discounting); Payback is 13 years. (c) Cash flow model for Well Technology B (initial flow rate at 0.6 bcf/y per well, Table 1) in shale gas field with 100 wells, assuming an initial gas price of $4/Mcf. (d) Cumulative cash flow) gives NPV = $591 million and IRR = 61% (both before discounting); payback is 4 years. Discounted net cash flow shows depreciated NPV further lowered by 10% discount.
5. Conclusions

Our conclusion is that cash flow models can be used to set targets for the rate of technology innovation required to make any particular shale play profitable. Technology optimization can add reserves by lowering well completion cost and improving the recovery factor of a particular field. For example, if a well’s EUR is doubled by improved hydraulic fracturing technology and well development cost is halved, this will improve cash flow more than a doubling of the wellhead gas price (Fig. 10). More effective production technology will not only increase the recovery factor, but also improves the $ Insert Table A1 here $.

Emerging shale gas plays typically have a high degree of subsurface uncertainty due to which field development in the early stage inevitably includes wells with a lower productivity and marginal cash flow. New and cheaper technology is needed to develop each well more effectively. The mean EUR for the field can grow when the project, or exactly that specific (constant) discount rate for which the NPV equals zero. In other words, solving $ NPV = 0 $ for $ CF $ gives the IRR.

The cash flow aggregated over the lifecycle of the project, also called the net present value (NPV), is given by the total, discounted, cumulative cash flow:

$$ NPV = \sum_{n=1}^{N} A_n (1 + CF_n)^{-n} $$

Here the years are denoted as $ n = 1, 2, \ldots, N $. The discounted internal rate of return (IRR) is the average rate of return over the lifecycle of the project, or exactly that specific (constant) discount rate for which the NPV equals zero. In other words, solving $ NPV = 0 $ for $ CF $ gives the IRR.

Table A1 shows the notation for each parameter.

### References


